Calculus2

Homework solution

A) Select the correct answer

1) The partial fraction decomposition of $\frac{1}{x^2(x^2+3)}$ is

a) $\frac{A}{x^2}+\frac{Bx+C}{x^2+3}$ ~~(b)~~ $\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+3}$ c) $\frac{A}{x^2}+\frac{B}{x^2+3}$ d) $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^2+3}$

2) $\int \frac{\cos(\ln x)}{x} dx$=

~~(a)~~ $\sin(\ln x)+c$ $b) -\sin(\ln x)+c$

$c) \ln x \cos(\ln x)+c$ $d) \cos(\ln x)+c$

3) If $(x, y) = (-1, -1)$ then the polar coordinates (r, θ) is equal to

a) $\left(\sqrt{2}, \frac{\pi}{4}\right)$ b) $\left(\sqrt{2}, \frac{3\pi}{4}\right)$ ~~(c)~~ $\left(\sqrt{2}, \frac{5\pi}{4}\right)$ d) $\left(\sqrt{2}, \frac{7\pi}{4}\right)$

4) The equation $r = 4 \sec \theta$ in polar coordinates represents a

a) Horizontal line $y = 4$

(~~b)~~ Vertical line $x = 4$

c) Circle with center $(0,2)$ and $r = 2$
d) Circle with center $(2,0)$ and $r = 2$

5) Only one of the following series diverges (See Quiz solution)

(a) $\sum \frac{5}{\sqrt[3]{n}}$ b) $\sum \frac{5}{n^{1.3}}$ c) $\sum \frac{5}{n^3+1}$ d) $\sum \frac{5}{n^{\sqrt{3}}}$

6) $\sum_0^\infty \frac{3}{2^n} = 3+\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\cdots = \frac{a}{1-r} = \frac{3}{1-\frac{1}{2}} = 6$

a) 3 b) $\frac{3}{2}$ c) 2 ~~(d)~~ 6

7) The sequence $\left\{\frac{2^n}{3^{n+1}}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty}\frac{2^n}{3^{n+1}} = \lim_{n\to\infty}\frac{2^n}{3^n * 3} = \lim_{n\to\infty}\left(\frac{2}{3}\right)^n * \frac{1}{3} = 0 \qquad \text{since} \qquad \lim_{n\to\infty} a^n = \begin{cases} 0 & 0<a<1 \\ \infty & a>1 \end{cases}$$

a) converge to 1 ~~(b)~~ **converge to 0** *c) converge to 2* *d) Diverge*

8) If the partial sum of $\sum_{n=1}^{\infty} a_n$ is $S_n = \frac{2n^2+3n+1}{n^2+5n+3}$, **then the series**

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\frac{2n^2+3n+1}{n^2+5n+3} = \lim_{n\to\infty}\frac{2n^2}{n^2} = 2$$

(Note: this is a series not a sequence)

a) converge to 3 *b) converge to 0* *c) diverge* ~~(d)~~ **converge to 2**

9) One of the following series converges

a) -3+3-3+3-3+… **=$\sum(-1)^n 3$**	**b)** $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots=\sum\frac{1}{n^2}$	$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots=\sum\frac{1}{n}$	$\sum \cos n$
Diverges by Divergent test $\lim_{n\to\infty}(-1)^n 3$ **D.N.E**	**Conv. P-series p=2** >1	**Diverge P-series p=1** ≤ 1	**Diverges by Divergent test** $\lim_{n\to\infty}\cos n$ **D.N.E**

10) The series $\sum_{1}^{\infty}\sqrt[n]{e}$ $\qquad \lim_{n\to\infty} e^{\frac{1}{n}} = e^0 = 1 \quad \neq 0$

a) *Convergent by p-series test* *b) Divergent by p-series test* **c) Diverges by Divergent test.**
d) Convergent by Geometric series test

11) The improper integral

$$\int_0^{\infty}\frac{1}{1+x^2}dx = \lim_{k\to\infty}\tan^{-1}x\Big|_0^k = \lim_{k\to\infty}\tan^{-1}k - \tan^{-1}0 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

a) Converge and equal $\frac{\pi}{2}$ *b) Converge and equal* $\frac{\pi}{4}$ *c) Converge and equal 0* *d) diverge*

12) $\sum_0^\infty \frac{(-2)^n}{n!} = e^{-2}$ (**Remember:** $\sum_0^\infty \frac{(x)^n}{n!} = e^x$)

a) e^2 b) $-2e$ ~~(c)~~ $\frac{1}{e^2}$ d) $\frac{1}{e^2} - 2$

B) **Fill in the blanks with the answers**

1) **The trigonometric substitution that solves the integral**

$\int \frac{1}{x\sqrt{4x^2+25}} dx$ **is** __$2x = 5\tan\theta$

2) $\int_1^\infty \frac{1}{x^5} dx =$

$$\lim_{k\to\infty} \int_1^k x^{-5}\, dx = \lim_{k\to\infty} \left.\frac{x^{-4}}{-4}\right|_1^k = \lim_{k\to\infty} \left.\frac{1}{-4x^4}\right|_1^k = \lim_{k\to\infty} \frac{1}{-4k^4} - \frac{1}{-4} = \frac{1}{4}$$

3) **Given that** $\frac{1}{1-x} = 1 + x + x^2 + \cdots \quad -1 < x < 1$**, the power series**

of $\frac{x}{4+x} = x\ \frac{1}{4\left(1-\left(-\frac{x}{4}\right)\right)} = \frac{x}{4}\sum_0^\infty \left(-\frac{x}{4}\right)^n = \frac{x}{4}\sum_0^\infty (-1)^n \left(\frac{x}{4}\right)^n = \sum_0^\infty (-1)^n \left(\frac{x}{4}\right)^{n+1}$

where $-1 < -\frac{x}{4} < 1$, hence $-4 < x < 4$